## Exercise 7

What is the volume of the parallelepiped with sides $2 \mathbf{i}+\mathbf{j}-\mathbf{k}, 5 \mathbf{i}-3 \mathbf{k}$, and $\mathbf{i}-2 \mathbf{j}+\mathbf{k}$ ?

## Solution

Label each of the sides as

$$
\begin{aligned}
& \mathbf{a}=(2,1,-1) \\
& \mathbf{b}=(5,0,-3) \\
& \mathbf{c}=(1,-2,1)
\end{aligned}
$$

The volume of the parallelepiped formed by these vectors is given by the triple product,

$$
\begin{aligned}
\mathbf{a} \cdot(\mathbf{b} \times \mathbf{c}) & =(2,1,-1) \cdot\left|\begin{array}{ccc}
\hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\
5 & 0 & -3 \\
1 & -2 & 1
\end{array}\right| \\
& =\left|\begin{array}{ccc}
2 & 1 & -1 \\
5 & 0 & -3 \\
1 & -2 & 1
\end{array}\right| \\
& =-5\left|\begin{array}{cc}
1 & -1 \\
-2 & 1
\end{array}\right|+0-(-3)\left|\begin{array}{cc}
2 & 1 \\
1 & -2
\end{array}\right| \\
& =-5[(1)(1)-(-1)(-2)]+3[(2)(-2)-(1)(1)] \\
& =-10,
\end{aligned}
$$

or rather its magnitude: $V=|\mathbf{a} \cdot(\mathbf{b} \times \mathbf{c})|=10$.

